CHAPTER 41

Elementary Particles and the Beginning of the Universe

1* · How are baryons and mesons similar? How are they different?

Similarities: Baryons and mesons are hadrons, i.e., they participate in strong interaction. Both are composed of quarks. Differences: Baryons consist of three quarks and are fermions. Mesons consist of two quarks and are bosons. Baryons have baryon number +1 or -1. Mesons have baryon number 0.

2 · The muon and the pion have nearly the same mass. How do these particles differ?

The muon is a lepton. It is a spin-1/2 particle; it is a fermion. It does not participate in strong interactions. It appears to be an elementary particle like the electron.

The pion is a meson. Its spin is 0; it is a boson. It does participate in strong interactions. It is composed of quarks.

3 · How can you tell whether a decay proceeds via the strong interaction or the weak interaction?

A decay process involving the strong interaction has a very short lifetime (~10^{-23} s); decay processes that proceed via the weak interaction have lifetimes of order 10^{-10} s.

4 · True or false:

(*a*) All baryons are hadrons.

(*b*) All hadrons are baryons.

(*a*) True (*b*) False

5* · True or false:

Mesons are spin-1/2 particles.

False

6 · Two pions at rest annihilate according to the reaction $p^+ + p^- \to g + g$. (*a*) Why must the energies of the two g rays be equal? (*b*) Find the energy of each g ray. (*c*) Find the wavelength of each g ray.

(a) The initial momentum is zero; therefore the final momentum must be zero. The momentum of a photon is E/c. To conserve both momentum and energy the two photons must have the same momentum magnitude, hence the same energy.

(*b*) Use Table 41-1 $E_g = 139.6$ MeV

(*c*) $l = 1240/E$ (E in MeV, l in fm) $l = 8.88$ fm

7 · Find the minimum energy of the photon needed for the following pair-production reactions:

(*a*) $g \to p^+ + p^-$, (*b*) $g \to \mathrm{p} + \mathrm{p}^-$, and (*c*) $g \to m^- + m^+$.

(*a*), (*b*), (*c*) $E = 2m_ic^2$; see Tables 41-1 and 41-3	(*a*) $E = 279.2$ MeV; (*b*) $E = 1877$ MeV; (*c*) $E = 211.3$ MeV

8 · State which of the decays or reactions that follow violate one or more of the conservation laws, and give the law or laws violated in each case: (*a*) $p^+ \rightarrow n + e^+ + \bar{n}_e$, (*b*) $n \rightarrow p^+ + p^-$, (*c*) $e^+ + e^- \rightarrow g$, (*d*) $p + p^- \rightarrow g + g$, and (*e*) $\bar{n}_e + p \rightarrow n + e^+$.

(*a*) Energy conservation is violated; $m_p < m_n$.

(*b*) Energy conservation is violated; $m_n < m_p + m_p$.

(*c*) Momentum conservation is violated; two (or more) g rays must be emitted to conserve momentum.

(*d*) This is an allowed process; none of the conservation laws are violated.

(*e*) This is an allowed process; the lepton number is -1 on both sides and energy is conserved.

9* · Determine the change in strangeness in each reaction that follows, and state whether the reaction can proceed via the strong interaction, the weak interaction, or not at all: (*a*) $\Omega^- \rightarrow \Xi^0 + p^-$, (*b*) $\Xi^0 \rightarrow p + p^- + p^0$, and (*c*) $\Lambda^0 \rightarrow p^+ + p^-$.

(*a*) 1. List S of Ω^-, Ξ^0, and p^-; see Fig. 41-2	Ω^-, $S = -3$; Ξ^0, $S = -2$; p^-, $S = 0$
2. Determine ΔS	$\Delta S = +1$; reaction via the weak interaction is allowed
(*b*) 1. List S of Ξ^0, p, p^-, and p^0	Ξ^0, $S = -2$; p, $S = 0$; p^-, $S = 0$; p^0, $S = 0$
2. Determine ΔS	$\Delta S = +2$; reaction is not allowed
(*c*) 1. List S of Λ^0, p^+, and p^-	Λ^0, $S = -1$; for p and p^-, $S = 0$
2. Determine ΔS	$\Delta S = +1$; reaction via the weak interaction is allowed

10 · Determine the change in strangeness for each decay, and state whether the decay can proceed via the strong interaction, the weak interaction, or not at all: (*a*) $\Omega^- \rightarrow \Lambda^0 + K^-$ and (*b*) $\Xi^0 \rightarrow p + p^-$.

(*a*) S changes from -3 to -2; $\Delta S = +1$; the reaction can proceed via the weak interaction.

(*b*) S changes from -2 to 0; $\Delta S = +2$; the reaction is not allowed.

11 · Determine the change in strangeness for each decay, and state whether the decay can proceed via the strong interaction, the weak interaction, or not at all: (*a*) $\Omega^- \rightarrow \Lambda^0 + \bar{n}_e + e^-$ and (*b*) $\Sigma^+ \rightarrow p + p^0$.

(*a*) S changes from -3 to -1; $\Delta S = +2$; the reaction is not allowed.

(*b*) S changes from -1 to 0; $\Delta S = +1$; the reaction can proceed via the weak interaction.

12 · (*a*) Which of the following decays of the t particle is possible?

$t \rightarrow m^- + \bar{n}_m + n_t$

$t \rightarrow m^- + n_m + \bar{n}_t$

(*b*) Explain why the other is not possible. (*c*) Calculate the kinetic energy of the decay products for the decay that is possible.

(*a*) The first decay is allowed. It satisfies energy conservation and conservation of both the t and m lepton numbers.

(*b*) The second decay scheme is not allowed; it does not conserve t and m lepton numbers.

(*c*) $K_{\text{tot}} = m_t c^2 - m_m c^2$; see Table 41-3	$K_{\text{tot}} = 1784 \text{ MeV} - 106 \text{ MeV} = 1678$ MeV

The kinetic energy of the individual decay products cannot be determined from the decay scheme alone.

Note: In the first printing of the textbook the first decay scheme reads, "$t \rightarrow m^- + \bar{n}_p + n_t$". In that case, neither of the two reactions is allowed; both violate conservation of lepton number (there is no $\bar{n}_p$).

13* ·· Consider the following decay chain:

$\Omega^- \rightarrow \Xi^0 + p^-$

$\Xi^0 \rightarrow \Sigma^+ + \mathrm{e}^- + \bar{n}_\mathrm{e}$

$p^- \rightarrow m^- + \bar{n}_m$

$\Sigma^+ \rightarrow \mathrm{n} + p^+$

$p^+ \rightarrow m^+ + n_m$

$m^+ \rightarrow \mathrm{e}^+ + \bar{n}_m + n_\mathrm{e}$

$m^- \rightarrow \mathrm{e}^- + \bar{n}_\mathrm{e} + n_m$

(*a*) Are all the final products shown stable? If not, finish the decay chain. (*b*) Write the overall decay reaction for Ω^- to the final products. (*c*) Check the overall decay reaction for the conservation of electric charge, baryon number, lepton number, and strangeness.

(*a*) No; the neutron is not stable: $\mathrm{n} \rightarrow \mathrm{p}^+ + \mathrm{e}^- + \bar{v}_\mathrm{e}$

(*b*) Adding the reactions one obtains $\Omega^- \rightarrow \mathrm{p}^+ + \mathrm{e}^+ + 3\mathrm{e}^- + v_\mathrm{e} + 3\bar{v}_\mathrm{e} + 2\bar{v}_m + 2v_m$

(*c*) 1. Charge conservation: -1 → 1 + 1 - 3 = -1; charge is conserved.

2. Baryon number: 1 → 1; baryon number is conserved.

3. Lepton number; electrons: 0 → -1 + 3 + 1 - 3 = 0; lepton number for electrons is conserved.

Lepton number; muons: 0 → -2 + 2 = 0; lepton number for muons is conserved.

4. Strangeness: -3 → 0; strangeness is not conserved. However, in each baryon decay $\Delta S = +1$, and each decay is allowed via the weak interaction.

14 ·· Test the following decays for violation of the conservation of energy, electric charge, baryon number, and lepton number: (*a*) $\mathrm{n} \rightarrow p^+ + p^- + m^+ + m^-$; (*b*) $p^0 \rightarrow \mathrm{e}^+ + \mathrm{e}^- + g$. Assume that linear and angular momentum are conserved. State which conservation laws (if any) are violated in each decay.

(*a*) 1. $m_n > 2m_p + 2m_m$; energy conservation is not violated.

2. The total charge is 0 on both sides; charge conservation is not violated.

3. Baryon number changes from +1 to 0; conservation of baryon number is violated.

4. Lepton number is 0 on both sides; lepton number is conserved.

The process is not allowed because it violates conservation of baryon number.

(*b*) 1. $m_p > 2m_e$; energy conservation is not violated.

2. The total charge is 0 on both sides; charge conservation is not violated.

3. Baryon number is 0 on both sides; conservation of baryon number is not violated.

4. Lepton number is zero on both sides; conservation of lepton number is not violated.

The process is allowed.

15 · How can you tell whether a particle is a meson or a baryon by looking at its quark content?

A meson has 2 quarks, a baryon has 3 quarks.

16 · Are there any quark–antiquark combinations that result in a nonintegral electric charge?

No; from Table 41-2 it is evident that any quark-antiquark combination always results in an integral or zero charge.

17* · Find the baryon number, charge, and strangeness for the following quark combinations and identify the hadron:

(*a*) *uud*, (*b*) *udd*, (*c*) *uus*, (*d*) *dds*, (*e*) *uss*, and (*f*) *dss*.

(*a*) - (*f*) For each quark combination determine the baryon number *B*, the charge *q*, and the strangeness *S*; then see Table 41-2 to find a match.

(*a*) $B = 1$, $q = +e$, $S = 0$; the hadron is p^{+}
(*b*) $B = 1$, $q = 0$, $S = 0$; the hadron is n
(*c*) $B = 1$, $q = +e$, $S = -1$; the hadron is Σ^{+}
(*d*) $B = 1$, $q = -e$, $S = -1$; the hadron is Σ^{-}
(*e*) $B = 1$, $q = 0$, $S = -2$; the hadron is Ξ^{0}
(*f*) $B = 1$, $q = -1$, $S = -2$; the hadron is Ξ^{-}

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18 · Repeat Problem 17 for the following quark combinations: (*a*) $u\bar{d}$, (*b*) $\bar{u}d$, (*c*) $u\bar{s}$, and (*d*) $\bar{u}s$.

(*a*), (*b*), (*c*), (*d*) Use Table 41-2

(*a*) $B = 0$, $Q = +1$, $S = 0$; p^{+}; (*b*) $B = 0$, $Q = -1$, $S = 0$; p^{-};
(*c*) $B = 0$, $Q = +1$, $S = +1$; K^{+}; (*d*) $B = 0$, $Q = -1$, $S = -1$; K^{-}

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19 · The Δ^{++} particle is a baryon that decays via the strong interaction. Its strangeness, charm, topness, and bottomness are all zero. What combination of quarks gives a particle with these properties?

From Table 41-2 we see that to satisfy the conditions of charge = +2 and zero strangeness, charm, topness, and bottomness, the quark combination must be *uuu*.

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20 · Find a possible combination of quarks that gives the correct values for electric charge, baryon number, and strangeness for (*a*) K^{+} and (*b*) K^{0}.

(*a*) See Problem 18(*c*); $K^{+} = us$

(*b*) For K^{0} we need zero charge, $B = 0$, and $S = +1$. The combination $d\bar{s}$ satisfies these conditions.

21* · The D^{+} meson has no strangeness, but it has charm of +1. (*a*) What is a possible quark combination that will give the correct properties for this particle? (*b*) Repeat (*a*) for the D^{-} meson, which is the antiparticle of the D^{+}.

(*a*) $B = 0$, so we must look for a combination of quark and antiquark. Since it has charm of +1, one of the quarks must be *c*. Since the charge is +*e*, the antiquark must be $\bar{d}$. The possible combination for D^{+} is $c\bar{d}$.

(*b*) Since D^{-} is the antiparticle of D^{+}, the quark combination is $\bar{c}d$.

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22 · Find a possible combination of quarks that gives the correct values for electric charge, baryon number, and strangeness for (*a*) K^{-} (the K^{-} is the antiparticle of the K^{+}) and (*b*) $\overline{K}^{0}$.

(*a*), (*b*) See Problem 20; use the antiquark combinations

(*a*) $K^{-} = \bar{u}s$; (*b*) $\overline{K}^{0} = \bar{d}s$

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23 ·· Find a possible quark combination for the following particles: (*a*) Λ^{0}, (*b*) p^{-}, and (*c*) Σ^{-}.

(*a*) For Λ^{0}, $B = +1$, $Q = 0$, and $S = -1$. The quark combination that satisfies these conditions is *uds*.

(*b*) For p^{-}, the proper combination is the antiquark system of Problem 17(*a*), i.e., $\bar{u}\bar{u}\bar{d}$.

(*c*) For Σ^{-}, $B = +1$, $Q = -1$, and $S = -1$. The quark combination that satisfies these conditions is *dds*.

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24 ·· Find a possible quark combination for the following particles: (*a*) $\bar{n}$, (*b*) Ξ^{0}, and (*c*) Σ^{+}.

(*a*) For $\bar{n}$, the proper combination is the antiquark system of Problem 17(*b*), i.e., $\bar{u}\bar{d}\bar{d}$.

(*b*) For Ξ^{0}, $B = +1$, $Q = 0$, and $S = -2$. The quark combination that satisfies these conditions is *uss*.

(*c*) Note that the Σ^{+} is *not* the antiparticle of the Σ^{-} (see Table 41-1). For Σ^{+}, $B = +1$, $Q = +1$, and $S = -1$.

The quark combination that satisfies these conditions is *uus*.

25* ·· Find a possible quark combination for the following particles: (*a*) Ω^- and (*b*) Ξ^-.

(*a*) For Ω^-; $B = +1$, $q = -e$, $S = -3$. The quark combination that meets these conditions is *sss*.

(*b*) For Ξ^-; $B = +1$, $q = -e$, $S = -2$. The quark combination that meets these conditions is *ssd*.

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26 ·· State the properties of the particles made up of the following quarks: (*a*) *ddd*, (*b*) $u\bar{c}$, (*c*) $u\bar{b}$, and (*d*) $\overline{sss}$.

(*a*), (*b*), (*c*), (*d*) See Table 41-2

(*a*) $B = +1$, $Q = -1$, $S = 0$; (*b*) $B = Q = S = 0$, charm = -1;
(*c*) $B = 0$, $Q = +1$, $S = 0$, bottomness = -1;
(*d*) $B = -1$, $Q = +1$, $S = +3$

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27 · True or false:

(*a*) Leptons consist of three quarks.

(*b*) The times for decays via the weak interaction are typically longer than those for decays via the strong interaction.

(*c*) Electrons interact with the proton via the strong interaction.

(*d*) Strangeness is not conserved in weak interactions.

(*e*) Neutrons have no charm.

(*a*) False (*b*) True (*c*) False (*d*) True (*e*) True

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28 · (*a*) What conditions are necessary for a particle and its antiparticle to be the same? Find the antiparticle for (*b*) p^0 and (*c*) Ξ^0.

(*a*) It must be a meson, and it must consist of a quark and its antiquark.

(*b*) The p^0 is its own antiparticle.

(*c*) The Ξ^0 is a baryon; it cannot be its own antiparticle; the antiparticle is the $\overline{\Xi}^0 = \bar{u}\,\bar{s}\,\bar{s}$.

29* ·· Consider the following decay chain:

$\Xi^0 \rightarrow \Lambda^0 + p^0$

$\Lambda^0 \rightarrow \mathrm{p} + p^-$

$p^0 \rightarrow g + g$

$p^- \rightarrow m^- + \bar{n}_m$

$m^- \rightarrow \mathrm{e}^- + \bar{n}_\mathrm{e} + n_m$

(*a*) Are all the final products shown stable? If not, finish the decay chain. (*b*) Write the overall decay reaction for Ξ^0 to the final products. (*c*) Check the overall decay reaction for the conservation of electric charge, baryon number, lepton number, and strangeness. (*d*) In the first step of the chain, could the Λ^0 have been a Σ^0?

(*a*) Yes, the final products are stable.

(*b*) The end result is $\Xi^0 \rightarrow p^+ + e^- + \bar{v}_\mathrm{e} + v_m + \bar{v}_m + 2g$.

(*c*) 1. Charge conservation: $0 \rightarrow e^+ + e^- = 0$; charge is conserved

2. Baryon number: $1 \rightarrow 1 + 0 = 1$; baryon number is conserved.

3. Strangeness: $-2 \rightarrow 0$; $\Delta S = +2$; the reaction is allowed via the weak interaction because in the first two decays $\Delta S = +1$.

(*d*) No; the rest masses of the decay products would be greater than the rest mass of the Ξ^0, violating energy conservation.

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30 ·· Test the following decays for violation of the conservation of energy, electric charge, baryon number, and lepton number: (*a*) $\Lambda^0 \rightarrow \mathrm{p} + p^-$, (*b*) $\Sigma^- \rightarrow \mathrm{n} + \mathrm{p}^-$, (*c*) $m^- \rightarrow \mathrm{e}^- + \bar{n}_{\mathrm{e}} + n_m$. Assume that linear and angular momentum are conserved. State which conservation laws (if any) are violated in each decay.

(*a*) $m_\Lambda > m_\mathrm{p} + m_p$; energy is conserved. There is no change in charge; charge is conserved. There is no change in *B*; baryon number is conserved. There is no change in lepton number; lepton number is conserved. The decay is allowed.

(*b*) $m_\Sigma < m_\mathrm{n} + m_\mathrm{p}$; energy is not conserved. Charge is conserved. *B* changes from +1 to 0; baryon number is not conserved. Lepton number is conserved. The decay violates energy and baryon number conservation; it is not allowed.

(*c*) This decay satisfies all conservation laws and is allowed. It is the decay process for the m^- (see Example 41-2).

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31 ··· (*a*) Calculate the total kinetic energy of the decay products for the decay $\Lambda^0 \rightarrow \mathrm{p} + p^-$. Assume the Λ^0 is initially at rest. (*b*) Find the ratio of the kinetic energy of the pion to the kinetic energy of the proton. (*c*) Find the kinetic energies of the proton and the pion for this decay.

(*a*) Use Table 41-1	$K_\mathrm{tot} = (1116 - 938.3 - 139.6)\ \mathrm{MeV} = 38.1\ \mathrm{MeV}$
(*b*) Use momentum conservation (nonrelativistic)	$v_p/v_\mathrm{p} = m_\mathrm{p}/m_p$; $K_p/K_\mathrm{p} = m_\mathrm{p}/m_p = 6.72$
(*c*) Use the result of part (*b*); $K_p + K_\mathrm{p} = K_\mathrm{tot}$	$K_p = 33.2\ \mathrm{MeV}$; $K_\mathrm{p} = 4.93\ \mathrm{MeV}$

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32 ··· A Σ^0 particle at rest decays into a Λ^0 plus a photon. (*a*) What is the total energy of the decay products? (*b*) Assuming that the kinetic energy of the Λ^0 is negligible compared with the energy of the photon, calculate the approximate momentum of the photon. (*c*) Use your result for (*b*) to calculate the kinetic energy of the Λ^0. (*d*) Use your result for (*c*) to obtain a better estimate of the momentum and the energy of the photon.

(*a*) Use Table 41-1	$E = 1193\ \mathrm{MeV}$
(*b*) $p = E_g/c$; $E_g = E - m_\Lambda c^2$	$p = (77\ \mathrm{MeV})/c$
(*c*) $p_\Lambda = p_g$; $K_\Lambda = p_\Lambda^2/2m_\Lambda$	$K_\Lambda = 2.66\ \mathrm{MeV}$
(*d*) $E_g = E - m_\Lambda c^2 - K_\Lambda$; $p = E_g/c$	$p_g = (74.3\ \mathrm{MeV})/c$

33* ··· In this problem, you will calculate the difference in the time of arrival of two neutrinos of different energy from a supernova that is 170,000 light-years away. Let the energies of the neutrinos be $E_1 = 20$ MeV and $E_2 = 5$ MeV, and assume that the rest mass of a neutrino is 20 eV/c^2. Because their total energy is so much greater than their rest energy, the neutrinos have speeds that are very nearly equal to *c* and energies that are approximately $E \approx pc$.

(*a*) If t_1 and t_2 are the times it takes for neutrinos of speeds u_1 and u_2 to travel a distance *x*, show that

$$\Delta t = t_1 - t_2 = x\frac{u_1 - u_2}{u_1 u_2} \approx \frac{x\,\Delta u}{c^2}.$$

(*b*) The speed of a neutrino of rest mass m_0 and total energy *E* can be found from

Equation 39-25. Show that when $E >> m_0c^2$, the speed *u* is given approximately by $\dfrac{u}{c} \approx 1 - \dfrac{1}{2}\left(\dfrac{m_0 c^2}{E}\right)^2$.

(*c*) Use the results for (*b*) to calculate $u_1 - u_2$ for the energies and rest mass given, and calculate Δt from the result for (*a*) for $x = 170{,}000c\cdot\mathrm{y}$. (*d*) Repeat the calculation in (*c*) using $m_0c^2 = 40$ eV for the rest energy of a neutrino.

(*a*) 1. Express $\Delta t = t_2 - t_1$ in terms of u_1 and u_2	$\Delta t = \frac{x}{u_2} - \frac{x}{u_1} = \frac{x(u_1 - u_2)}{u_1 u_2}$
2. Note that $u_1 u_2 \approx c^2$; let $\Delta u = u_1 - u_2$	$\Delta t = \frac{x \Delta u}{c^2}$
(*b*) 1. Use Equ. 39-25 to write u/c	$u/c = \sqrt{1-(m_0 c^2/E)^2}$
2. Use the binomial expansion; $(m_0 c^2/E)^2 << 1$	$u/c \approx 1 - 1/2(m_0 c^2/E)^2$
(*c*) 1. Write $u_1 - u_2$ in terms of E_1 and E_2 and $m_0 c^2$	$u_1 - u_2 =$ ‰$c(m_0 c^2)^2\left(\frac{1}{E_2^2} - \frac{1}{E_1^2}\right) =$
2. Evaluate Δt for $m_0 c^2 = 20$ eV, $E_1 = 20$ MeV,	$\frac{c(m_0 c^2)^2(E_1^2 - E_2^2)}{2 E_1^2 E_2^2}$
$E_2 = 5$ MeV, and $x = 170{,}000\ c\cdot$y	$\Delta u = 7.5\times10^{-12}c$; $\Delta t = 1.275\times10^{-6}$ y $= 40.2$ s
(*d*) Repeat (*c*), part 2 for $m_0 c^2 = 40$ eV	$\Delta u = 4\times7.5\times10^{-12}c = 3\times10^{-11}c$; $\Delta t = 4\times40.2$ s $= 161$ s

Note that the spread in the arrival time for neutrinos from a supernova can be used to estimate the rest mass of a neutrino.