An Empirical Re-Examination of the Hotelling Valuation Principle

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Abstract

The Hotelling Principle states that the unit price of an exhaustible resource less the marginal cost of extracting it will tend to rise over time at a rate of interest equal to the return on comparable capital assets. Using an extended data set and Ordinary Least Squares (OLS) regression techniques, we produced cross-sectional results in accordance with the predictions of the Hotelling Principle. Recent literature casts great doubt on the use of time series analysis to uncover the economic principle.

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1 Introduction

More than 60 years ago, Harold Hotelling proposed his theory of exhaustible natural resource pricing. This theory, called the Hotelling Principle, states that the unit price of an exhaustible resource less the marginal cost of extracting it (or the net price) will tend to rise over time at a rate of interest equal to the return on comparable capital assets. Hotelling focused mainly on the normative issues involved in the debate, stating that the "invisible hand" of the marketplace would induce the current generation of resource owners to use up just the right amount of the resource and to conserve the rest for future generations. Hotelling's fundamental insight has been the basis of virtually all normative economic analysis dealing with natural resource policy.

Many empirical estimations of the Hotelling Principle have typically used direct tests of the relationship between interest rates and the growth rate of prices of particular minerals over various periods of time. The findings of these studies have not been strongly supportive of the Hotelling Principle. One factor that has likely contributed to these adverse findings is that over time, other variables such as improvements in technology serve to mitigate the effects of intertemporal scarcity. Using data over more than half of the twentieth century, Smith (1981) found support for arbitrage behavior linking resource markets with markets for other capital assets that could cloud the Hotelling model over time. In addition, these findings should be viewed with suspicion given the fact that historical prices are not very accurate and contain considerable measurement error.

2 Theoretical Discussions of Hotelling's Theories

Gordon (1994) analyzes Hotelling's theories and their implications as to current policy debates. In relation to the subject at hand, the author calls into question a few common assumptions. First, the assumption that exploration immediately produces accurate estimates of recoverable deposits is flawed; instead, the extraction of iron ore and copper supplies may be delayed as a result of a variety of circumstances including the development of specialized technology unique to each situation. Second, the assumption that substantial exhaustion effects exist may also be incorrect. Gordon cites the explanation given by Barnett and Morse (Barnett and Morse (1963)) in determining that ingenuity and profit incentive will drive firms to devise technology that will overcome the effects of natural resource exhaustion. In support of this claim, he cites empirical work done by M.A. Adelman and Campbell Watkins (Adelman and Watkins (1995)), demonstrating that mineral values do not increase over time. The author accepts the belief that such ingenuity and innovation will continue. Clearly, further advances will occur and result in several decades of continued progress, justifying continued optimism regarding natural resource use.

3 Empirical Work Based on Hotelling's Theories

Denise Young (Young (1991)) examines the operation of Canadian copper producers in order to determine if their behavior is consistent with the Hotelling conditions. This paper modifies the Hotelling Principle into a two-staged model in which the firm must make two interrelated decisions. Since the metal mining industry is typically characterized by situations in which many minerals may be extracted from a single ore, the owner must decide (1) how much ore to extract and (2) how much of each mineral to remove from each extracted ore. The Hotelling Principle implies that the discounted marginal cost of the last unit of ore extracted in any period will be equal to that of any other period. Based on this type of situation, the marginal revenue from selling the last unit of a particular mineral must be equal to the marginal cost of producing that unit.

In Young's analysis, the data consisted of a cross-section of annual observations on output, costs, and other pertinent variables for 14 Canadian copper-mining firms during the period 1955–1981. After estimating a cost function independent of the corresponding Hotelling requirements or Euler equations, the Hotelling conditions were imposed on the cost function. The entire set of parameters being analyzed then became insignificant. Whether the cost function is estimated separately from or in conjunction with the cost function, the results are not consistent with the underlying theory. The author noted that this may be due to an incorrect specification of the cost function or other underlying functions as well as the omission of information regarding the entry and exit of firms into the industry.

Merton Miller and Charles Upton (Miller and Upton (1985)), proposed an alternative testing strategy based on an alternative implication of Hotelling's analysis. In particular, they showed that in a world in which the time path of mineral prices, less marginal extraction costs, follows Hotelling's principle, the value of the reserves in any currently operating, optimally managed mineral deposit depends mainly on current prices and extraction costs, regardless of when the reserves are extracted. Miller and Upton tested this proposition, which they call the Hotelling Valuation Principle, by regressing the per unit market values of the reserves of a sample of U.S. domestic oil and gas producing companies on the difference between the price and the extraction cost Hotelling values at several points in time during the years 1979–1981. The Miller and Upton results indicated that Hotelling values can account for a substantial portion of the variation in market values of the firms in the sample.

In another attempt to evaluate the Hotelling Principle, G.C. Watkins (Watkins (1992)) examined 27 reserve transactions in Canada covering the time period 1989–1991. The information consisted of the transaction value and volume of reserves. This data is then used in conjunction with the Hotelling Valuation Principle in an effort to test its empirical relevance. This approach offers the advantage of deriving a measure of the reserve value of a natural resource by using the actual transaction prices of the resource *in situ*. The results, based on this sample, are contrary to those of Miller and Upton and to Hotelling's theory in general. Further, in 1995 Adelman and Watkins expanded this analysis, to include transactions that took place in the fourth quarter of 1990. With the same specifications, the results of analyzing these 34 transaction were once again in conflict with Miller and Upton's results. The authors note that such results do not undermine or negate the validity of Hotelling's theorem. They point out that various features of the industry, such as exploration, technological change, cost relationships, and investment cycles, serve to complicate the comparison of the empirical data and theory.

This paper is based largely on Miller and Upton's work as well as the work of David May (May (1995)) and the specifications used therein. The goal is to test the Hotelling Valuation Principle using a different and somewhat expanded data set, to determine if similar results could be produced.

4 A Mathematical Restatement of the Hotelling Pricing Principle

Consider a profit-maximizing, price-taking owner of an exhaustible resource at time zero. The reserves may be extracted either in the current period or in any of the next N periods. The extraction costs at time t are $C_t = C_t(q_t, Q_t)$, where q_t is the rate of extraction at time t and the cumulative level of extraction is indicated as

$$Q_t = \sum_{s=0}^{t} q_s \tag{1}$$

We know that $\frac{\partial C_t}{\partial Q_t}$ is non-negative and will be positive if additional reserves are increasingly expensive to extract. The discounted present value of profits is then

$$V_o = \sum_{t=0}^{N} \frac{p_t q_t - C\left(q_t, Q_t\right)}{\left(1+r\right)^t} \tag{2}$$

where p_t is the expected market price of output at time t, the rate of interest is r, which is assumed to be known and constant over time, and N is a known date beyond which production can safely be presumed to have ceased. V_o is maximized subject to the constraint

$$\sum_{t=0}^{N} q_t \leq R_o \tag{3}$$

where R_o are total reserves. This expression is written as an inequality because reserves are in principle an economic quantity and it need not be optimal to extract all of the reserves. Assuming that it is optimal to recover all of the reserves, the first-order condition for profit maximization in any period is

$$(p_t - c_t)\left(\frac{1}{1+r}\right)^t - \sum_{s=t}^{N}\left(\frac{\partial C_s}{\partial Q_s}\right)\left(\frac{1}{1+r}\right)^s = \lambda;\ t = 0, \cdots, N \tag{4}$$

where C_t is the marginal cost of extraction in period t and λ is the Lagrange multiplier on the constraint.

To simplify further, consider the case of a production function with extraction costs per unit of output independent of cumulative output so that $\frac{\partial C_t}{\partial Q_t} = 0$. Then the first order condition is simply

$$(p_t - c_t)\left(\frac{1}{1+r}\right)^t = \lambda;\ \ t = 0, \cdots, N \tag{4$'$}$$

Solving this system of difference equations yields the familiar Hotelling Principle:

$$(p_t - c_t) = (p_o - c_o)(1+r)^t;\ t = 0,\ \cdots, N \tag{5}$$

This equation states that the efficient extraction over time of an exhaustible resource implies that the real price of that resource, net of its marginal extraction costs, grows at a rate equal to the real rate of interest.

5 The Hotelling Valuation Principle

Under the further assumption of constant returns to scale in current as well as cumulative extraction, the valuation expression (2) takes a simple form. Marginal cost is the same as average cost so that substitution of (5) into (2) yields as the present value of total reserves

$$V_o = (p_o - c_o\sum_{t=0}^{N} q_t = (p_o - c_o)R_o \tag{6}$$

Thus, where output prices, net of extraction costs, obey the Hotelling principle, the value of the total reserves in any mineral property depends solely on the current price per unit of the mineral, net of current extraction costs. Units of production in future years will earn higher net prices, but in a Hotelling world, the present value of the net price of any unit must be the same, regardless of when extracted. Expression (6) is a special case of what Miller and Upton call the Hotelling Valuation Principle. The valuation principle can be empirically tested by regressing observed market values per unit of reserves at a given point in time on the current output prices net of marginal extraction costs. The equation to be empirically evaluated is

$$\frac{V_o^{it}}{R_o^{it}} = \alpha + \beta(p_o^{it} - c_o^{it}) \tag{7}$$

where i indexes companies, t indexes time, and o signifies the then current values for the variables as of the sample date t.

If the Hotelling Valuation Principle is valid, its test relies on the values of the coefficients α and β. Under the constant returns to scale assumption, we would expect $\alpha = 0$ and $\beta = 1$ and also that any other variable added to the equation should not add any explanatory power to (7).

6 Extensions of the Valuation Principle

Two extensions to the Valuation Principle mentioned by Miller and Upton are noteworthy. In the first extension, Miller and Upton explain that diminishing returns to scale in current production will impact only the constant term, but not the slope. If diminishing returns to scale are present, the intercept term α in the proposed regression can no longer be presumed to be zero under the Hotelling Valuation Principle, but the slope coefficient β remains unity.

The second extension of the Valuation Principle concerns itself with the scenario of when extraction costs are allowed to rise over time along with cumulative production. Miller and Upton state that the time path of net prices rise at a rate less than the real rate of interest r of the Hotelling Principle. Nevertheless, Miller and Upton show that the slope coefficient is still one and the intercept will be less than zero.

7 Empirical Test of the Hotelling Valuation Principle

Miller and Upton, in their econometric analysis, used data from 39 firms over the years 1979 to 1981. It should also be noted that a cross-sectional analysis was conducted by David May which examined 22 firms in the year 1991 (May (1995)). In a similar manner to both the Miller and Upton study and the May study, we examine the operations of firms in the oil and gas industry. The cross-sectional and time-series data consists of 90 observations of 15 companies over the years 1986–1995.

The source of the data was the *Oil and Gas Journal*, which publishes a listing of the companies with the largest volume of oil and natural gas reserves. Many of these firms were not included in the sample because their operations were too diverse, including substantial revenues and expenses unrelated to the extraction of oil and natural gas, or because there was an absence of adequate financial information. Thus, the sample consists of firms with activities limited primarily to exploration and production as compared to more vertically integrated companies with transportation, refining, and retailing operations. *Moody's Industrial Manual* provides financial information pertaining to both the number of outstanding shares and the market value of those shares as well as revenue and expense information. Furthermore, some additional observations were excluded based on any indication by *Moody's* that a company experienced an extraordinary event in a certain year, such as entry into or exit from the industry during the fiscal year.

In this paper, two series of empirical tests are performed to address the application of the Hotelling Valuation Principle to the data. The first series involves an evaluation basing all calculations on a conversion rate of 5.7mcf of natural $gas = 1$ barrel of oil equivalents (boe), which is the conversion utilized in both the Miller and Upton paper (Miller and Upton (1985)) and the May paper (May (1995)). The dependent variable is $\frac{V}{R}$, which is the market value of the company divided by the total reserves. The independent variable is the current net price per unit which is given by $(p_o - c_o)$ and will be referred to as the HOTEL variable. The second regression equation in this subset retains the original dependent variable $\frac{V}{R}$ as well as the independent variable, HOTEL, but adds an additional independent variable, YEAR, which will attempt to capture changes in the slope parameter through time. The third equation in this subset also retains the original variables $\frac{V}{R}$ and HOTEL, but in this case a different independent variable, GULF, is added in order to capture any effects that the Persian Gulf War had on the market value of companies within this industry, presumably impacting the slope parameter. The GULF variable was assigned a value of one for observations from both 1990 and 1991 when the Gulf War might have had an effect and a value of zero for observations from all other years.

The second series of tests was based on an alternative conversion rate as proposed by G.C. Watkins in his analysis of Canadian reserve transactions. Specifically, he suggests and utilizes a conversion rate of 12.0 mcf of natural gas = 1 boe due to the higher per unit transportation costs associated with natural gas. All other aspects of these regression equations are identical to the first subset of equations. See Table 3 for a description of the various variables used in this paper.

8 Regression Results

As indicated in Table 1, in the first model ordinary least squares (OLS) regression results reveal a constant term with a value of 0.10; however, it is insignificant. Under the original Hotelling Valuation Principle hypothesis, the value of the coefficient on HOTEL should be one. Employing a Wald Coefficient Test, the null hypothesis that HOTEL = 1 is not rejected1.

As can be seen in Table 1, throughout the first subset of regression equations the results were essentially similar. In each case, a Wald Coefficient Test employing the null hypothesis that HOTEL = 1 is not rejected at any significance level. The intercept term was not significant at any level. Of note, the YEAR variable was significant at the 5% level and positive, indicating an increase in the valuation of the reserves through time. The GULF variable carried a negative coefficient and was significant at only the 10% level. This might suggest that financial markets exhibited some lack of confidence in the oil industry during the time period of the Gulf War. Overall, these results are in agreement with the Hotelling Valuation Principle and support those obtained in the Miller and Upton study and that of May (1995).

Table 1 : Regression Results

Conversion	5.7 mcf of natural gas = 1 boe			12.0 mcf of natural gas = 1 boe		
Regression	(1)	(2)	(3)	(4)	(5)	(6)
Hotel	0.953	0.948	0.967	0.914	0.894	0.917
F-Statistic	0.133	0.169	0.068	0.596	0.932	0.453
Probability	0.716	0.682	0.794	0.442	0.337	0.453
Intercept	0.100	−0.288	0.206	0.425	−0.148	0.633
	(0.18)	(−0.50)	(0.38)	(0.57)	(−0.19)	(0.86)
Year	-	0.099**	-	-	0.169***	-
		(1.88)			(2.14)	
Gulf War	-	-	−0.725*	-	-	−1.024*
			(−1.98)			(−1.86)
Adjusted R-squared	0.375	0.393	0.395	0.429	0.450	0.444

Dependent variable = market value of oil and *gas/barrel* of oil equivalents. $N = 90$ and t values are in parentheses. Wald Test was performed with respect to Hotel = 1; This test was the source of the F-Statistic and probabilities. *** significant at the 1% level; ** significant at the 5% level; * significant at the 10% level

In contrast, when the conversion rate was changed to reflect the higher per unit transportation costs associated with natural gas, the results are very similar to the results of these previous works. As before, the HOTEL coefficient was subjected to a Wald Coefficient Test where the null hypothesis tested is HOTEL = 1, and in each case, the null hypothesis is not rejected. While this is consistent with the theory, it should be noted that this may be a more acceptable conversion rate to use as indicated by the adjusted R-Squared which increased in each case. The remaining results were largely the same as those taken from the first subset of equations. One notable difference was that the YEAR coefficient was slightly larger and significant at the 1% level as opposed to only the 5% level.

As stated above, the Hotelling Valuation Principle predicts that the coefficient on the constant term is expected to be zero and the value of the coefficient on HOTEL should be one. Two additional sets of regressions were estimated to determine if a proportional relationship exists between the dependent variable $\frac{V}{R}$ and the independent variable HO-

TEL. The exact same regressions that were previously estimated were now estimated using ordinary least squares (OLS) except that the intercept term was removed in order to "force" the regression line through the origin.

The results are summarized in Table 2, and reflect the use of different conversion rates. In the first set of results, where 5.7mcf of natural gas = 1 barrel of oil equivalents (boe), the independent variable HOTEL is once again subjected to a Wald Coefficeint Test. The null hypothesis that HOTEL = 1 is not rejected. These results would seem to confirm the validity of the Hotelling Valuation Principle using this particular data set.

Table 2 : Regression Results

Conversion	5.7 mcf of natural gas = 1 boe			12.0 mcf of natural gas = 1 boe		
Regression	(1)	(2)	(3)	(4)	(5)	(6)
Hotel	0.976	0.891	0.952	0.975	0.876	0.909
F-Statistic	0.436	3.43	0.355	0.535	4.917	1.475
Probability	0.511	0.067*	0.533	0.466	0.029**	0.228
Intercept	-	-	-	-	-	-
Year	-	0.089*	0.052	-	0.164**	0.132
		(1.83)	(0.88)		(2.22)	(1.49)
Gulf War	-	-	−0.493	-	-	−0.426
			(−1.12)			(−0.66)
Adjusted R-squared	0.392	0.397	0.399	0.432	0.456	0.453

Dependent variable = market value of oil and gas/barrel of oil equivalents. $N = 90$ and t values are in parentheses. Wald Test was performed with respect to Hotel = 1; This test was the source of the F-Statistic and probabilities. *** significant at the 1% level; ** significant at the 5% level; * significant at the 10% level

Additional regressions were estimated once again using $\frac{V}{R}$ as the dependent variable, but in these specifications of the model, the independent variables YEAR and GULF were added. The independent variable YEAR is added to capture any changes in the slope parameter through time, and GULF is added to capture any effects that the Persian Gulf War had on the market value of companies within this industry. When the independent variable YEAR is added to the model, it's coefficient is 0.089 and is significant at the 10% level. However, when a Wald Coefficient Test is performed under the null hypothesis that HOTEL = 1, the null hypothesis is rejected at the 10% level of significance. In the specification including both independent variables YEAR and GULF, neither is significant.

The second set of results use the alternative conversion rate, where 12.0 mcf of natural gas = 1 boe and are also summarized in Table 2. The dependent variable is once again $\frac{V}{R}$ and the independent variable in the first specification is HOTEL. The results using this conversion rate are quite similar to the previous results, with two exceptions. The coefficient HOTEL is once again tested using the Wald Coefficient Test under the null hypothesis that HOTEL = 1. The hypothesis is rejected in only one other specification, namely regression result 5 in Table 2. These results also are in accordance with the predictions of the Hotelling Valuation Principle, with the two exceptions noted.

Similarly, for this conversion rate, additional regressions were estimated using the dependent variable $\frac{V}{R}$ and the independent variables YEAR and GULF. The coefficient on YEAR is 0.164, which is significant at the 5% level. The HOTEL variable, under the null hypothesis that HOTEL = 1 and using the Wald Coefficient Test, indicates that the null is rejected for both of these specifications. When the independent variable GULF is added to the specification, its coefficient insignificant in both specifications.

Overall, the results from this specification of the model where the intercept term is removed so as to "force" the regression line through the origin would seem to validate the Hotelling Valuation Principle for this particular data set.

Miller and Upton demonstrate that a difference in the value of the HOTEL coefficient and one is not likely to be related to non-constant returns to scale, which, if present, would affect only the constant term, not the slope. A more feasible theoretical explanation would be that an element of uncertainty exists. The Hotelling principle states that the value of natural resource reserves is independent of location. That is, a barrel of oil that has been extracted is of equal value to the same amount of oil *in situ*. However, a potential investor may be uncertain that a firm will be able to continue to extract the oil at an equivalent net price per barrel. Thus, a rational investor would tend to discount the value of those reserves in situ due to uncertainty.

Of course, another explanation for the small difference in the value

Table 3 : Variable Names and Descriptions

V	=	shares of common and preferred stock outstanding multiplied by FYE stock price
R	=	oil and natural gas reserves at FYE converted to barrel of oil equivalents (boe)
p	=	oil and natural gas revenues divided by volume of oil and natural gas production expressed in boe
c	=	extraction costs divided by volume of oil and natural gas production expressed in boe
$HOTEL$	=	current net price per unit
$YEAR$	=	year indicator
$GULF$	=	Persian Gulf War indicator 1 if year is 1990 and 1991 and 0 otherwise
V/R	=	market value of the company divided by the total reserves

of the HOTEL coefficient and the statistically insignificant constant term might simply be the limitation of the data. In addition, measurement error may exist in terms of the financial information for each of the firms, which was not presented in a completely uniform manner by *Moody's*. Difficulties might also arise from the use of year end stock prices to value each company. As these prices reflect a specific point in time only, it might be possible that an extraordinary event, not related to a firm's operations, could bias the market's valuation of that firm on that day. Further, as noted in the Watkins paper, a more ideal measure of the reserve value of such a resource may be obtained by using the actual reserve transaction prices, but these values are usually private information.

Note, that for all equations in which the YEAR variable was included, the regression exhibited some element of heteroskedasticity as indicated by a White test. In these particular cases, this was corrected by means of a heteroskedasticity consistent covariance matrix. In all of the other specifications, no heteroskedastic error was present.

9 Conclusion

The Hotelling Valuation Principle states simply that the per unit value of reserves in the ground is the same as the current value above the ground less the marginal extraction cost. This is true because the expected trend of future net product prices and the market discount rate will exactly offset each other through time. While our data set was not exhaustive, with an extended sample the results we obtained do correspond relatively closely to this theory and to the results obtained by Miller and Upton and May. The minimal difference in the slope coefficient on HOTEL, which is slightly less than unity, may be attributable to the element of uncertainty and risk. Young and Ryan (1996) incorporate a risk premia into the industry level Hotelling model without completely reconciling such a model with historical price and cost data. This is the same finding when risk is incorporated at the firm level by Slade and Thille (1995). Future research might focus on the incorporation of the element of uncertainty in order to achieve more accurate result and obtain further understanding of the Hotelling Valuation Principle as it applies to various situations found in the real world, but we are lead to conclude that the Hotelling principle can only be isolated at the cross-sectional level so far in natural resource economics.

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